Multiple equilibria in a Dynamic Mating Game with Discrete Types and Similarity Preferences

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Abstract

We consider the dynamic decentralised non atomic mating game Γ_n over n periods, initially presented by Alpern and Reyniers (1999). We are dealing especially with the two period mutual choice game $\Gamma_2(m)$, where individuals can have m types.

In the Alpern and Reyniers game, two populations are randomly matched for n periods. Players have one dimensional types which are uniformly distributed over a continuous or a discrete interval. There exist a continuum of players and no new players can enter the game in any period. In each period, each party of a matched pair (i,j) can either accept or reject the other. If both accept, then they form a mated couple and leave the game, with both paying a cost of |i-j|. Otherwise, they both proceed unmated into the next period. This process is called 'mutual choice' selection. At the end of the game, all players prefer to be mated than to remain unmated. Players have similarity preferences, searching for a partner whose type is close to their own. Hence, they try to minimise their cost of mating, defined above as the absolute distance between their type and the type of their potential partner.

In the current paper, we present briefly the analysis of the continuous-type Γ_n game and focus on the discrete-type $\Gamma_n(m)$ game. Our main result is the existence of multiple equilibria in $\Gamma_n(m)$ which contrasts with the analysis of Alpern and Reyniers and the relevant literature, since in the latter only one equilibrium is described. Moreover, we provide a method for determining all the possible equilibria in the discrete-type game. Finally we comment on the effectiveness and stability of the equilibrium strategies in the game $\Gamma_2(m)$.

key words: mating game, mutual mate choice, similarity preferences

1 Introduction

We focus on the analysis of dynamic decentralised games of mate search. Most of the work in this paper has its background in the dynamic non atomic mating games Γ_n and $\Gamma_n(m)$ introduced by Alpern and Reyniers [1]. In the game $\Gamma_n(m)$ that we examine, two equal populations X and Y are randomly paired over n successive periods. For simplicity, we call the population of X males and the population of Y females. All players have one-dimensional types, uniformly distributed in a discrete symmetric interval $I_m = \{-m, -m+1, ..., 0, ...m-1, m\}$. Hence there is a finite number of types, but it is assumed that there exist a continuum of players for each type. The positive and the negative types have symmetric preferences and behaviour and therefore the interval of the types is taken to be symmetric around the origin 0. All types in the last period prefer to be mated than to remain unmated.

In each period, each party of a matched pair (i, j) can either accept or reject the other. If both accept, then they form a mated couple and leave the game, with both paying a cost of $|i - j|$. Otherwise, they both proceed unmated into the next period. This process is called 'mutual choice' selection. In the last period, all players prefer to be mated rather than remain unmated. No new players can enter the game in any period.

The game Γ_n has the same properties as $\Gamma_n(m)$, but in Γ_n players have one-dimensional types uniformly distributed in a continuous and not a discrete interval.

In both Γ_n and $\Gamma_n(m)$ games, the type and the preferences of each player affect his/her strategy and his/her expected utility from a matching. All players are thought to have similarity preferences; therefore they prefer mates of a type similar to their own. They aim at minimising their cost from a pairing, hence the distance between their types. Similarity preferences appear frequently in real life [1], [2], [4], [3], [5] and [10], and their analysis has applications in several settings, such as the mating of animals or humans, the matching of tennis players, the matching of sellers and buyers of houses etc.

In the described $\Gamma_n(m)$ game, every pairing has the same cost for both the matched male and female. Hence if a type i male is paired with a type j female, the mating cost for both i and j is $|i - j|$. Nevertheless, it is not always true that when a female (male) accepts a male (female), the male (female) will also accept the female (male). Whether a type i accepts another type j in some period, depends on the average cost that i expects to receive in the following periods. It is important to note that the expected cost differs for each type, since the type distribution changes in every period. As a result, it is possible that a type i is choosier in a period than an other type j and not willing to accept the latter, even though j is willing to be mated with i. In general, central types are choosier, since they have more types close to them and consequently more chances to meet someone close to their own type.

Given the players' cost functions, Alpern and Reyniers use a notion of equilibrium similar to Subgame Perfect Nash equilibrium, finding a Subgame Perfect Nash equilibrium in a continuum of players; at equilibrium every player accepts those types which provide him/her with a smaller cost than his/her expected cost in the next period. We define equilibrium in the same way, and in section 2 we present the results of Alpern and Reyniers' analysis.

In [1] and in the relevant literature, it seems to be implicitly assumed that there exists a unique equilibrium, since in the examples examined only one equilibrium is presented. The basic contribution of this paper is to prove the existence of multiple equilibria in some discrete games. Specifically, it is found that when there are $m = 3$ or $m = 5$ positive types (and respectively 3 or 5 symmetric negative types and a 0 type), there exist 3 equilibrium strategies in the two period game. Computer search, which was not exhaustive, shows that the existence of multiple equilibria is possible also when the number of periods is larger. For example, it was found that at least two equilibria exist in the 4 period model when there are $m = 8$ positive types (and respectively 8 symmetric negative types and a 0 type). Furthermore, this paper offers a method for finding all the possible equilibrium strategies in the discrete game $\Gamma_2(m)$.

The applications of both Γ_n and $\Gamma_n(m)$ are multiple in fields such as biology, evolution, economics, politics and social sciences. In biology, mate choice is one of the basic areas of sexual selection and several game theoretical models exist which try to predict animal behaviour, such as the models described in [3] and in [9]. Furthermore, there are various cases in economics and social sciences where decentralised matching is involved, such as the the matching of buyers and sellers of cars or finding a partner to chat with at a cocktail party [7]. Besides, mating games can also be applied on other problems such as the matching of firms and underwriters [8] or the two sided "marriage" model [6]. Our goal is to explore further the area of dynamic mate choice and to contribute to the analysis of real life problems.

The paper is organised as follows. In the first section we illustrate the symmetric continuous game Γ_n of Alpern and Reyniers. Next, we focus on the corresponding discrete game $\Gamma_n(m)$, where we assume having m positive types, m negative types and a type 0. We describe a method of finding all the equilibrium strategies in $\Gamma_2(m)$, and then we present our numerical results for the two period game when there are $m = 1$ to $m = 9$ positive types, analysing the equilibrium strategies, their effectiveness (in reducing mean intra-couple type difference) and their stability . Finally, we discuss the existence of multiple equilibria in games of n periods, where $n > 2$, and give an example of the existence of multiple equilibria in the 4 period game $\Gamma_4(m)$.

2 Description of the Non Atomic Mating Game Γ_n

In the Γ_n game, two identical populations of individuals, of known one dimensional types, are uniformly distributed in an interval. We take this interval to be $[-1, 1]$. Individuals have similarity preferences as described before (preferring potential partners having types close to their own) and the distance between the individuals' types $|x - y|$ is a measure of the cost of a mating of an individual of type x with an individual of type y. We assume that there exist a continuum of players.

In every period, individuals are randomly paired and they form a couple if they both accept each other; otherwise they move on to the next period, where a random matching takes place again. The process continues until period n, where all matchings are mutually accepted and form mated couples.

A strategy is defined as an acceptance rule indicating the maximum distance $s_k(x)$ acceptable in period k to a type x individual. That is, a type x male will accept a type y female in period k if

$$s_k(x) \geq |x - y| \tag{1}$$

Both sexes use same strategies in this symmetric model. Hence a type x male is going to use the same strategy $s_k(x)$ with a type x female in every period k.

When the population is uniformly distributed between $[-1, 1]$, types x and $-x$ share the same strategy.

$$s_k(x) = s_k(-x) \tag{2}$$

It follows from (1) that in period k, given that type x uses strategy $s_k(x)$ and type y uses strategy $s_k(y)$, a mating occurs if and only if

$$|x - y| \leq \min\{s_k(x), s_k(y)\} \tag{3}$$

The expected cost for an individual of type x to enter in the k^{th} period unmated is denoted by $c_k(x)$; this depends on strategy s, so $c_k(x) = c_k(s, x)$.

At equilibrium, no player can decrease his/her expected cost by using other than the equilibrium strategy $\hat{s}$, given that the rest of the population is using strategy $\hat{s}$. Hence at equilibrium

$$\hat{s}_k(x) = c_{k+1}(\hat{s}, x), \ \ for\ 1 \leq k \leq n - 1\ and\ -1 \leq x \leq 1 \tag{4}$$

If F is the normalised cumulative probability function of types y in the last period, the expected cost for a type x entering in the last period is

$$c_n(x) = \int_{-1}^{1} |x - y|\, dF(y)$$

Before presenting the $\Gamma_n(m)$ game and illustrating our results, it is important to mention two theorems of Alpern and Reyniers, which are important for our analysis. In these theorems it is assumed that F can take any form, hence the theorems can be applied also in the case that F is a point function as in $\Gamma_n(m)$.

Theorem 1 (of [1]) *If F denotes the final period cumulative probability distribution, then the final period cost function c_n is a symmetric convex function, minimised at 0, with the following properties:*

$$c_n(-1) = c_n(1) = 1 \tag{5}$$

$$c'_n(-1) = -1,\ c'_n(1) = 1 \text{ and } c'_n(0) = 0 \tag{6}$$

$$c'_n(x) = 2F(x) - 1 \tag{7}$$

Theorem 2 (of [1]) *In a two period model, at equilibrium, if $x_0 < x_1 < x_2$, and x_0 accepts x_2, then x_1 also accepts x_2.*

Since c_n is increasing in $|x|$ by Theorem 1, the choosier individuals tend to be nearer to the centre. As we move away from the centre, types become less choosy, with the extreme types accepting every type between themselves and the middle types. This fact, combined with Theorem 2, shows that in the two period model, central types tend to do better than extreme types, as they have more chances to be accepted in the first period.

3 Discrete Type 2-Period Model $\Gamma_2(m)$

In the discrete type game $\Gamma_n(m)$, as it is described by Alpern and Reyniers, it is assumed that there exist $2m+1$ uniformly distributed types in the first period but that there exist a continuum of players of each type; in the beginning of the first period the fraction of the population having any type i is $\frac{1}{2m+1}$. In order to keep the symmetry of the continuous model, every type i belongs in the set $I_m = \{-m, -m+1, -m+2, \ldots, -2, -1, 0, 1, 2, \ldots, m-2, m-1, m\}$; a type i corresponds to a type $\frac{i}{m}$ in the continuous model.

The requirement of mutual acceptance is maintained in the discrete type model and the definition of a strategy s remains unchanged; $s_k(i)$ denotes the maximum distance that any type can have from i in order to be accepted by the latter in the k^{th} period; it is important to note though that in the discrete type model, a strategy $s_k(i)$ is always an integer. In the two period game $\Gamma_2(m)$, for simplicity reasons, since a strategy is defined only in the first period and a expected cost function is defined only in the second period, $s(i) = s_1(i)$ will denote the strategy of a type i in the first period and $c(i) = c_2(i)$ will denote the cost that a type i expects to get for entering in the last period unmated.

At equilibrium, the strategy of a type i in $\Gamma_2(m)$ is equal to the floor function of the expected cost for i in the last period. Hence

$$s(i) = \lfloor c(i, s) \rfloor \tag{8}$$

where $c(i,s)$ is the cost that i expects to get if it enters in the second period unmated, when s strategy is used.

As a result, even though two types may not have the same expected cost for entering unmated in the last period, they can have the same strategy. For example, if $c(i,s) = 2.1$ and $c(j,s) = 2.9$, then $s(i) = s(j) = 2$, for an equilibrium strategy s.

4 Properties of Equilibrium Strategies in $\Gamma_2(m)$

Alpern and Reyniers found an equilibrium strategy in $\Gamma_2(m)$ by seeking fixed points of a best response function on the strategy space. In other words, starting with any strategy s, they used the iterative method in order to find a new strategy $\tilde{s}$, whose value function has the same floor as $\tilde{s}$ for all types. This iterative procedure in general may miss some equilibria - especially the ones corresponding to repelling fixed points. To ensure that we find all the equilibria, we must adopt a more thorough method; hence for m positive types, it is adequate to check all $(m+1)^{m+1}$ possible strategies, examining which ones satisfy the equilibrium condition (8). However, as the number m of types increases, this method becomes extremely time consuming and the need to rule out some of the strategies becomes apparent. Thus, we use Theorem 1, in order to reduce the number of eligible equilibrium strategies.

Before enumerating the properties that an equilibrium strategy has, it is important to stress that even if the expected cost function c is strictly convex, it does not follow that its floor approximation strategy s is also strictly convex. This is due to the fact that a strategy $s(i)$ of a type i is always an integer, but the corresponding cost $c(i)$ does not have to be an integer and it can be larger than $s(i)$; $s(i) \leq c(i) < s(i)+1$. For example, a cost function $c = (1.99, 2.1, 2.4, 2.9)$ is strictly convex, while its corresponding strategy $s = (1, 2, 2, 2)$ is not. However, we will show, that s must be "almost convex", in senses that we make precise in Theorem 3.

Lemma 3 *If $s(i) - s(j) \geq 2$, then $c(i) - c(j) > 1$.*

Proof. By (8), $c(i) \geq s(i)$ and $c(j) < s(j) + 1$.

Hence $c(i) - c(j) > s(i) - s(j) - 1 \geq 1$. ■

Lemma 4 *If $c(i) - c(j) \geq 1$, then $s(i) - s(j) \geq 1$.*

Proof. *By (8), $c(i) < s(i) + 1$ and $c(j) \geq s(j)$.*

Hence $s(i) - s(j) > c(i) - 1 - c(j) \geq 0$.

Since $s(i)$ and $s(j)$ are both integers, it follows that $s(i) - s(j) \geq 1$. ■

Given Lemmas 3 and 4, we will determine the properties of the floor function of a convex function.

Theorem 5 *Let $s = \lfloor c \rfloor$ be an equilibrium strategy of $\Gamma_2(m)$, with corresponding cost function c. Let $i \geq 0$. s is of the form $s = (s(0), s(1), s(2), s(3), ..., s(m))$, where $s(i)$ denotes the strategy that a type i uses at equilibrium. Then,*

(i) $s(m) = m$

The extreme types (m and $-m$) must be universal acceptors.

Thus, for example when we have 5 types, $s = (1, 1, 1, 1, 1)$ cannot be a potential equilibrium strategy, while $\widetilde{s} = (1, 1, 1, 1, 5)$ may be.

(ii) $s(i + 1) \geq s(i)$, $\forall i$.

A strategy s has to be a non decreasing list.

For example, a list of the form $(1, 2, 1, 3, 4, 5)$ is not an equilibrium strategy since type 1 ($s(1) = 2$) accepts more types than type 3 ($s(3) = 1$).

(iii) If $s(i+1) - s(i) \geq 2$ *then* $s(i+2) - s(i+1) \geq s(i+1) - s(i) + 1$, $\forall i$.

For example, the following lists $(1, 2, 4, 4, 5, 5)$ and $(1, 3, 6, 7, 7, 7, 7, 7)$ are not possible equilibrium strategies.

(iv) If $s(i + 2) \geq s(i) + 2$ *then* $s(i + 4) > s(i + 2)$, $\forall i$.

Proof. (i) This is an immediate consequence of Theorem 1, as $s(m) = \lfloor v(m) \rfloor = m$ according to (5). Same way, for $-m$.

(ii) From (5) and (8), s is the non decreasing floor function of a non decreasing floor function, hence non decreasing.

(iii) Suppose that s is an equilibrium strategy, violating this condition. Then for some $l \geq 2$, we have $s(i) = a$, $s(i+1) = a+l$ and $s(i+2) = a+2l-2$. For strategy s to be an equilibrium strategy then from Theorem 1, it has to be true that

$$c(i + 1) - c(i) \leq c(i + 2) - c(i + 1)$$

By (8), we have

$$\begin{aligned} &c(i) \ \epsilon \ [a, a + 1) \\ &c(i + 1) \ \epsilon \ [a + l, a + l + 1) \\ &c(i + 2) \ \epsilon \ [a + 2l - 2, a + 2l - 1) \end{aligned}$$

Consequently

$$\begin{gathered} l - 1 < c(i + 1) - c(i) < l + 1 \\ \text{and} \\ l - 3 < c(i + 2) - c(i + 1) < l - 1 \end{gathered}$$

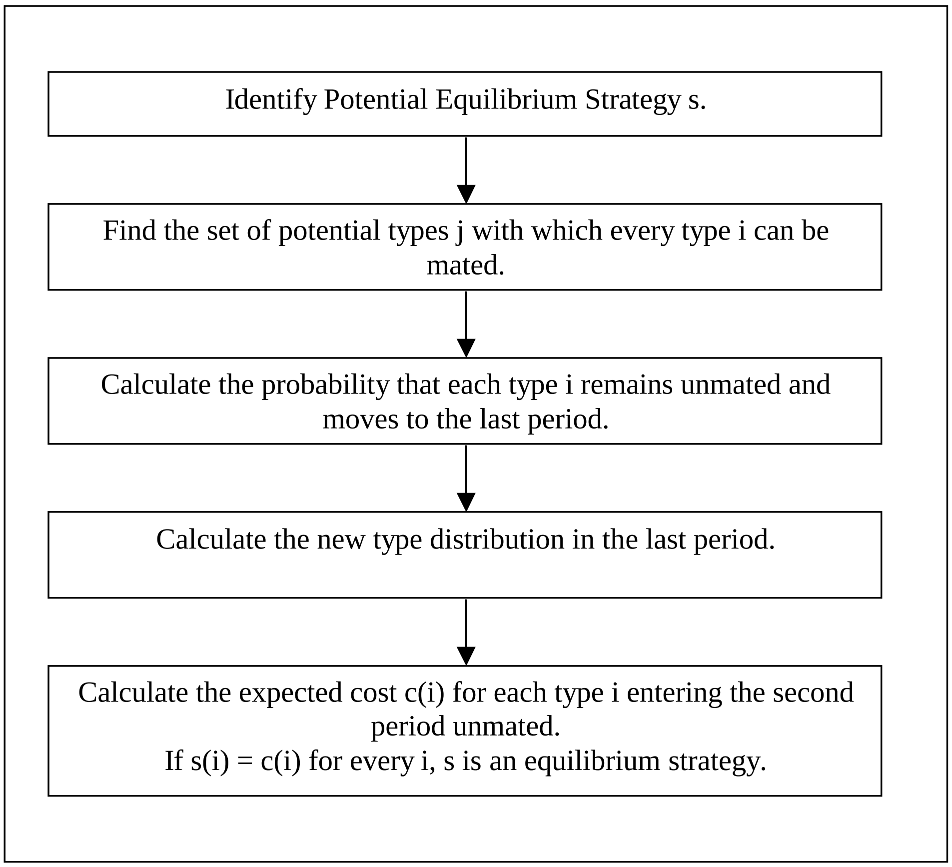
Hence $c(i+1)-c(i) > c(i+2)-c(i+1)$, which contradicts our initial claim.

(iv) By theorem 1, $c(i + 4) - c(i + 2) \geq c(i + 2) - c(i)$. But by Lemma 3, if $s(i + 2) \geq s(i) + 2$ then $c(i + 2) - c(i) > 1$.

Hence $c(i + 4) - c(i + 2) > 1$ and by Lemma 4 $s(i + 4) - s(i + 2) \geq 1$ hence $s(i + 4) > s(i + 2)$. ■

Having determined the properties of the equilibrium strategies, it is then easy to identify the potential equilibrium strategies and check which ones satisfy the equilibrium condition (8). In order to do so, given a potential equilibrium strategy s, we first have to find the set of potential types j with which every type i can be mated according to that strategy. Then, given the initial type distribution, we can calculate the probability of each type i remaining unmated and moving to the last period. This probability helps us next to determine the new type distribution in the last period and consequently the expected cost $c(i)$ for each type i entering the second period unmated; if $s(i) = \lfloor c(i) \rfloor$ for every type i, then s is proved to be an equilibrium strategy.

Table 1: Algorithm for identifying equilibrium strategies



5 Analysis of the Two Period Discrete Type Game $\Gamma_2(m)$ with $m = 0$ to $m = 9$ Positive Types

We used theorem 6 to identify all the potential equilibrium strategies in $\Gamma_2(m)$ for $m = 0$ to $m = 9$ positive types and then by applying the algorithm described in Table 1, we managed to identify all the existing equilibria in these games. Next we calculated the probability to get mated in the first period if the equilibrium strategies are used, and the expected cost for any type i. Finally we computed the intra-couple correlation and the inter-couple stability in each case.

The most important result of this analysis is the existence of 3 equilibrium strategies when there are $m = 3$ and $m = 5$ positive types, since multiple equilibria had not been noted in $\Gamma_2(m)$ till now. However, this analysis also helps us to get a better insight of the game itself. Connections between the strategy, the probability to be mated and the mating cost can be made and it is shown that correlation is not affected by the number of types in the game, while the inter-couple stability seems to be inversely correlated with the number of types.

5.1 Equilibrium Strategies

In all the examined games there exist an odd number of equilibria, ranging from 1 to 3, a fact that is in accordance with general theory. When there are $m = 1, 2, 4, 6, 7, 8, 9$ positive types there exists a unique equilibrium strategy. Nevertheless, when there are $m = 3$ and $m = 5$ positive types, there exist 3 equilibrium strategies. The equilibrium strategies for all the games are illustrated in table 2 and in plot 1.

In table 2, we list all the equilibrium strategies for $m = 1$ to 9, denoting the maximum distance each type $i = 0, 1, ..., m$ will accept in the first period. For example, when $m = 4$, at equilibrium $s(1) = 2$, hence type 1 accepts types $-1, 0, 1, 2$ and 3 and type -1 accepts types $-3, -2, -1, 0, 1$.

Plot 1 illustrates the expected cost $c(i)$ that each type i will pay if it enters in the last period unmated when the equilibrium strategy is used, for all examined m. The floor functions of c are the equilibrium strategies but c may give some extra information in the case where there are multiple equilibria. The plotted expected costs c are divided with m, so that $c(m) = 1$; by normalising c, it is easier to make comparisons when m varies.

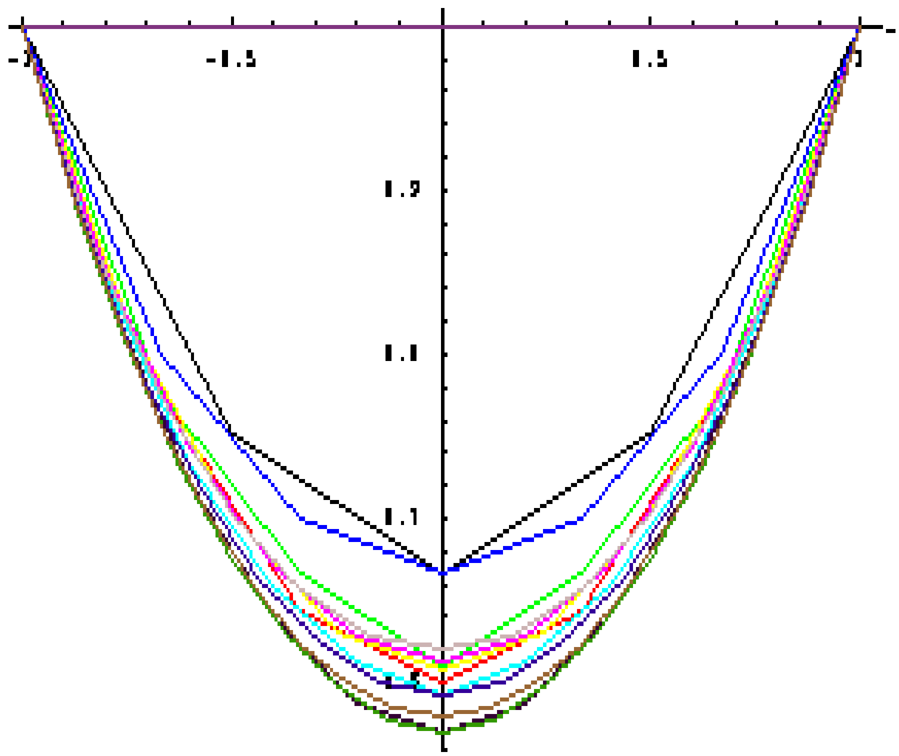
By looking at table 2 and at plot 1, it becomes obvious that in every game the middle type (type 0) is the choosiest type. As we move from the middle type (type 0) to the extreme types (m and $-m$), types become less choosy. The extreme types (m and $-m$) are the least choosy, accepting every type in all the cases. Moreover, it is evident that as the number of types increases, the graphs of the strategies seem to approach each other, a fact that suggests that for very large m we may have a unique equilibrium.

Plots 2 and 3 show the 3 equilibrium strategies when $m = 3$ and when $m = 5$ respectively, named $s1$, $s2$ and $s3$ in table 2. The equilibrium strategies in each case differ only on the strategies used by the middle types (close to 0), while the strategies used by the more extreme types (closer to m and $-m$) are the same. Nevertheless, even when the strategies are the same for some types i, where $i \neq m, -m$ (Table 2) the expected cost for entering in the last period unmated differs (Plots 1, 2 and 3).

Table 2. Equilibrium strategies in the first period, for $m = 1$ to 9.

		type0	type1	type2	type3	type4	type5	type6	type7	type8	type9
m=1		1	1								
m=2		1	1	2							
m=3	s1	**1**	**1**	**2**	**3**						
	s2	**1**	**2**	**2**	**3**						
	s3	**2**	**2**	**2**	**3**						
m=4		2	2	2	3	4					
m=5	s1	**2**	**2**	**3**	**3**	**4**	**5**				
	s2	**2**	**3**	**3**	**3**	**4**	**5**				
	s3	**3**	**3**	**3**	**3**	**4**	**5**				
m=6		3	3	3	4	4	5	6			
m=7		4	4	4	4	5	5	6	7		
m=8		4	4	4	5	5	5	6	7	8	
m=9		5	5	5	5	5	6	6	7	8	9

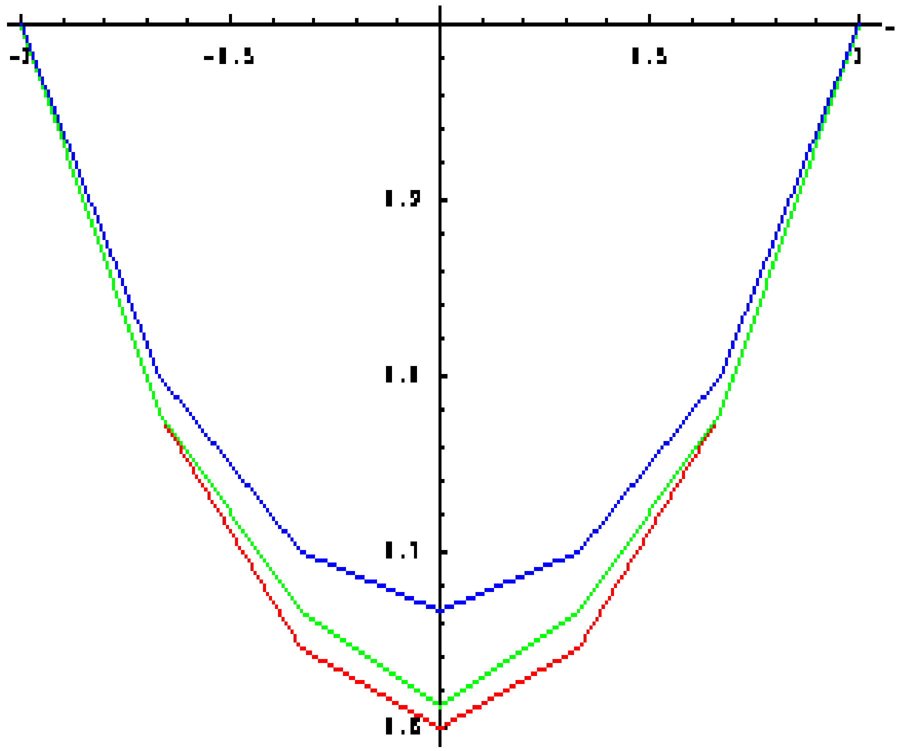
Plot 1. Normalised expected costs of entering in the last period unmated, when the equilibrium srategies are used, for m 1 to 9.



$m=1$	$m=2$	$m=3, s_1$	$m=3, s_2$	$m=3, s_3$	$m=4$	$m=5, s_1$	$m=5, s_2$
purple	black	red	light green	blue	yellow	dark grey	light blue

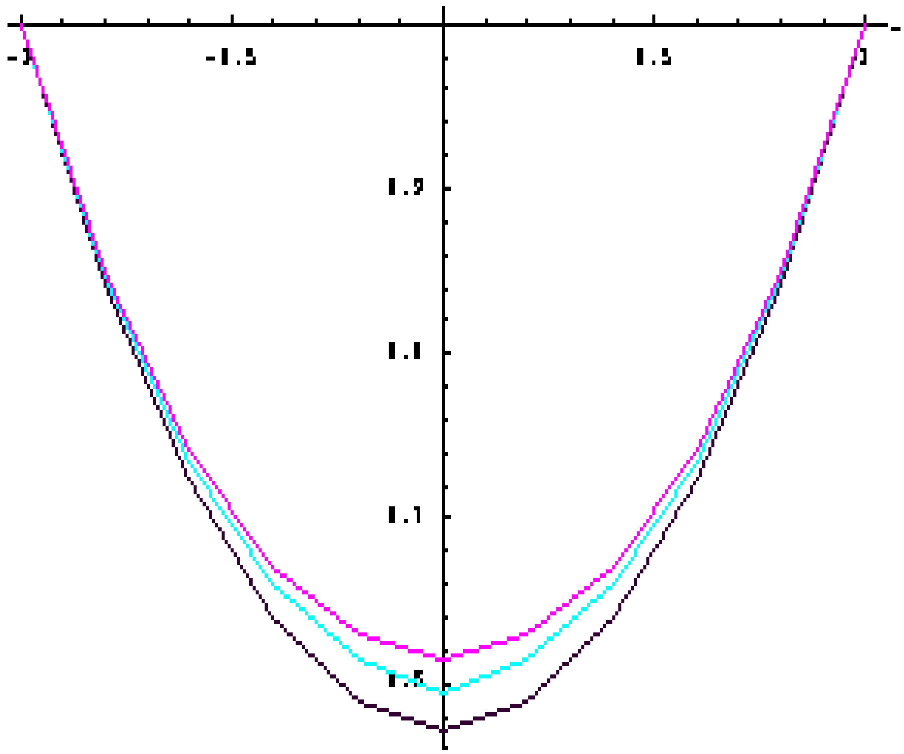
$m=5, s_3$	$m=6$	$m=7$	$m=8$	$m=9$
pink	light grey	dark blue	dark green	brown

Plot 2. Normalised expected costs of entering in the last period unmated, when the equilibrium strategies are used, for m 3.



s_1: red, s_2: light green and s_3: blue

Plot 3. Normalised expected costs of entering in the last period unmated, when the equilibrium strategies are used, for m 5.



s_1: dark grey, s_2: light blue and s_3: pink

5.2 Probability of Getting Mated in the First Period

In many cases, it is important to explore how the probability of getting mated in the first period changes, depending on the type and on m. In that way, we can estimate which types we expect to find a partner more quickly. Table 3 illustrates the probability for each type to be mated in the first period. It is obvious that the types closer to the middle (type 0) always have higher probability to be mated in the first period (higher than 0.5 in most cases except for $m = 5$ and $m = 3$), while the more extreme types (close to m and $-m$) have lower chances to find a mate in the initial period (lower than 0.5 in every case, except when $m = 1$). Additionally, it is apparent that in the cases where there are multiple equilibria, when the middle types are less choosy it is more probable to be mated in the first period, while whether the extreme types have a smaller chance of being mated, depends not only on the behaviour of the middle types (type 0), but also on the behaviour of the types between the extreme (m and $-m$) and the middle (type 0).

Table 3. Probability of getting mated in the first period, when the equilibrium strategy is used

		type0	type1	type2	type3	type4	type5	type6	type7	type8	type9
m=1		1	0.667								
m=2		0.6	0.6	0.4							
	s1	0.429	0.429	0.429	0.286						
m=3	s2	0.429	0.714	0.429	0.429						
	s3	0.714	0.714	0.571	0.429						
m=4		0.556	0.556	0.556	0.444	0.333					
	s1	0.455	0.455	0.545	0.455	0.364	0.364				
m=5	s2	0.455	0.636	0.636	0.455	0.455	0.364				
	s3	0.636	0.636	0.636	0.545	0.455	0.364				
m=6		0.538	0.538	0.538	0.538	0.462	0.385	0.308			
m=7		0.6	0.6	0.6	0.6	0.533	0.467	0.4	0.467		
m=8		0.529	0.529	0.529	0.588	0.529	0.471	0.412	0.353	0.353	
m=9		0.579	0.579	0.579	0.579	0.579	0.526	0.474	0.421	0.368	0.316

5.3 Total Expected Cost

For each equilibrium strategy found in Table 2, individuals of each type i have an expected total cost $C(i) = C(i, s)$ for entering the game, that is the expected distance $|i - j|$ from their eventual mate j. The total expected cost $C(i)$ for a type i, depends on the probability $p_1(i)$ of i being mated in the first period, the expected cost $d_1(i)$ of i being mated in first period and the expected cost $d_2(i)$ of i being mated in second period. Hence

$$C(i) = p_1(i)d_1(i) + (1 - p_1(i))d_2(i)$$

It is expected that types having a greater probability of being mated in the first period will have a bigger cost in the first period, but probably a smaller total cost. Therefore, since the probability of getting mated in the first period is higher for the middle types (type 0), they have the lowest cost of mating in total, but a large cost in the first period. The total mating cost tends to increase as we move from the middle types (type 0) to the extreme types (m and $-m$), which is probably explained by the fact that the mating probability in the first period tends to decrease as the types increase. Furthermore, the difference between the cost in two periods increases as we move from the middle types (type 0), to the extreme types (m and $-m$) and as we add more types in the game.

In the case of multiple equilibria, table 4 shows which strategy is preferred by each type, since it illustrates which strategy has the smallest expected total cost for a type. Thus, for $m = 3$, it becomes obvious that $s1 = (1, 1, 2, 3)$ is better for middle types -1, 0 and 1, $s2 = (1, 2, 2, 3)$ is better for types -2 and 2 and both $s2 = (1, 2, 2, 3)$ and $s3 = (2, 2, 2, 3)$ are equally good for extreme types 3 and -3. For $m = 5$, $s1 = (2, 2, 3, 3, 4, 5)$ should be preferred by -2.-1,0,1 and 2, $s2 = (2, 3, 3, 3, 4, 5)$ should be preferred by -4 and 4 and $s1 = (3, 3, 3, 3, 4, 5)$ should be favoured by -3 and 3, while the extreme types -5 and 5 should be indifferent between the equilibrium strategies.

Table 4. Total expected cost when the equilibrium strategy is used

			type0	type1	type2	type3	type4	type5	type6	type7	type8	type9	average
m=1		1rst period	0.667	0.333									0.444
		2nd period	0	0.333									0.222
		total	0.667	0.667									0.667
m=2		1rst period	0.4	0.4	0.2								0.32
		2nd period	0.533	0.6	1.2								0.827
		total	0.933	1	1.4								1.147
m=3	s1	1rst period	0.286	0.286	0.286	0.143							0.245
		2nd period	1.029	1.105	1.333	2.143							1.456
		total	1.314	1.39	1.619	2.286							1.701
	s2	1rst period	0.286	0.857	0.286	0.429							0.49
		2nd period	1.048	0.571	1.333	1.714							1.184
		total	1.333	1.429	1.619	2.143							1.673
	s3	1rst period	0.857	0.857	0.571	0.429							0.653
		2nd period	0.571	0.6	1.029	1.714							1.037
		total	1.429	1.457	1.6	2.143							1.69
m=4		1rst period	0.667	0.667	0.667	0.444	0.333	0.545					0.543
		2nd period	1.079	1.122	1.249	1.825	2.667	3.182					1.645
		total	1.746	1.788	1.915	2.27	3	3.727					2.188
m=5	s1	1rst period	0.545	0.545	0.818	0.545	0.364	0.545					0.562
		2nd period	1.556	1.604	1.457	1.973	2.676	3.182					2.122
		total	2.102	2.15	2.275	2.519	3.04	3.727					2.684
	s2	1rst period	0.545	1.09	1.09	0.545	0.636	0.545					0.76
		2nd period	1.618	1.115	1.2	2	2.309	3.182					1.93
		total	2.164	2.206	2.291	2.545	2.945	3.727					2.69
	s3	1rst period	1.09	1.09	1.09	0.818	0.636	0.545					0.86
		2nd period	1.117	1.143	1.221	1.688	2.318	3.182					1.838
		total	2.208	2.234	2.312	2.506	2.955	3.727					2.698
m=6		1rst period	0.923	0.923	0.923	0.923	0.692	0.538	0.462				0.757
		2nd period	1.62	1.651	1.744	1.897	2.465	3.2	4.154				2.449
		total	2.544	2.574	2.667	2.82	3.157	3.738	4.615				3.207
m=7		1rst period	1.333	1.333	1.333	1.333	1.067	0.867	0.733	0.667			1.067
		2nd period	1.658	1.68	1.745	1.855	2.342	2.948	3.709	4.667			2.637
		total	2.992	3.013	3.079	3.188	3.408	3.815	4.442	5.333			3.703
m=8		1rst period	1.176	1.176	1.176	1.47	1.176	0.941	0.765	0.647	0.882		1.038
		2nd period	2.149	2.173	2.248	2.075	2.539	3.1	3.785	4.623	5.176		3.152
		total	3.325	3.35	3.424	3.546	3.715	4.041	4.55	5.27	6.059		4.19
m=9		1rst period	1.579	1.579	1.579	1.579	1.579	1.316	1.105	0.947	0.842	0.789	1.274
		2nd period	2.198	2.216	2.272	2.364	2.494	2.993	3.586	4.294	5.143	6.158	3.433
		total	3.777	3.795	3.851	3.943	4.073	4.309	4.691	5.242	5.985	6.947	4.708

5.4 Intra-Couple Correlation

We are also interested in analysing the intra-couple correlation, since it shows how alike mated couples are. In order to find the intra-couple correlation coefficient, we have to find first the fraction of couples (i, j) in periods 1 and 2.

FIRST PERIOD

Given that i,j accept each other, the fraction of couples (i, j) created in the first period is $\frac{1}{2m+1}\frac{1}{2m+1}$. Initially,the fraction of individuals of type i is $\frac{1}{2m+1}$ and the fraction of individuals of type j is $\frac{1}{2m+1}$. Hence, $\frac{1}{2m+1}$ of the individuals of type i are mated with an individual of type j.

SECOND PERIOD

Given that strategy s is used, we define as $q(i, s)$ the probability that an individual of type i remains unmated in the first period and enters in the second period. Hence in the second period, there are $\frac{1}{2m+1}q(i, s)$ individuals of type i and $\frac{1}{2m+1}q(j, s)$ individuals of type j. Every individual accepts anyone he/she is paired with in the last period. Hence the probability of a type i being mated with a type j is $\frac{q(j,s)}{\sum_{k=-m}^{k=m} q(k,s)}$. Thus, the proportion of couples (i, j) in the second period when strategy s is used is

$$\frac{1}{2m+1}q(i, s)\frac{q(j, s)}{\sum_{k=-m}^{k=m} q(k, s)}$$

We define the function

$$a(s, i, j) = \begin{cases} 0, & \text{if } i \text{ and } j \text{ are not mated in the first period, hence if } |i - j| > s(i) \text{ or } |i - j| > s(j) \\ 1, & \text{if } i \text{ and } j \text{ are mated in the first period, hence if } |i - j| \leq s(i) \text{ and } |i - j| \leq s(j) \end{cases}$$

Hence the total fraction $n(s, i, j)$ of couples (i, j) in the first and second period, given that strategy s is used is

$$n(s, i, j) = a(s, i, j)\left(\frac{1}{2m+1}\right)^2 + \frac{1}{2m+1}q(i, s)\frac{q(j, s)}{\sum_{k=-m}^{k=m} q(k, s)} \tag{9}$$

Knowing the fraction of couples (i, j) in every period, it is easy then to calculate the correlation coefficient r.

The intra-couple correlation r seems to be quite small in all the games analysed, taking values between 0.306 and 0.36, probably due to the fact that all types are willing to accept a range of types including theirs. There are no significant differences between the different games or in the same game between different equilibria. r would probably grow with the number of periods n.

Table 5. Intra-couple correlation coefficient r, when the equilibrium strategy is used

	m=2	m=3	m=4	m=5	m=6	m=7	m=8	m=9
s1	0.36	0.306	0.337	0.34	0.33	0.34	0.34	0.336
s2		0.357		0.347				
s3		0.357		0.347				

5.5 Instability of Created Couples

Often, it is important to know how stable are the couples created given a specific strategy used. We need to know how willing the players would be to change partners after the end of the game. Assuming still that in every couple (i, j) created, i represents a male and j a female, we choose two couples (i, j) and (k, l) at random, where i and k are males and j and l are females. We define instability as the probability that either i prefers l better than j and l prefers i better than k, or that j prefers k better than i and k prefers j better than l.

We define the function w, such as

$$w(s,(i,j),(k,l)) = \begin{cases} 1, & \left\{ \begin{array}{c} \text{if } i \text{ prefers } l \text{ than } j \text{ and } l \text{ prefers } i \text{ than } k, \\ \text{hence if } \; |i-j| > |i-l| \; \text{ and } \; |l-k| > |i-l| \\ \textit{or} \\ \text{if } j \text{ prefers } k \text{ than } i \text{ and } k \text{ prefers } j \text{ than } l, \\ \text{hence if } \; |i-j| > |j-k| \; \text{ and } \; |k-l| > |k-j| \end{array} \right. \\ 0, & \text{otherwise} \end{cases}$$

Given that initially we have fixed cohorts of males and females, the population of each type of male or female is infinite and the probability of choosing 2 random couples (i, j) and (k, l) is $n(s, i, j)n(s, k, l)$ from (9). Hence

$$\textit{instability}(s) = \sum_{i=-m}^{i=m} \sum_{j=-m}^{j=m} \sum_{k=-m}^{k=m} \sum_{l=-m}^{l=m} w(s,(i,j),(k,l))n(s,i,j)n(s,k,l)$$

It becomes apparent from table 6, that as we add more types to the game, the couples created become more unstable. Focusing on the games with multiple equilibria, we can observe that there are no important differences between the different strategies. In any case, when $m = 3$, $s_1 = (1, 1, 2, 3)$ seems to be the most stable strategy and $s_3 = (2, 2, 2, 3)$ the most unstable and when $m = 5$,

$s_1 = (2,2,3,3,4,5)$ seems to be the most stable strategy and $s_3 = (3,3,3,3,4,5)$ the most unstable.

Table 6. Instability index

	m=2	m=3	m=4	m=5	m=6	m=7	m=8	m=9
s1	0.181	0.20634	0.24468	0.25714	0.2753	0.29297	0.29574	0.30691
s2		0.22326		0.26672				
s3		0.23819		0.27213				

6 Multiple Equilibria in an n-Period Game $\Gamma_n(m)$, where $n > 2$

In the two period discrete games analysed, we were able to find all the equilibrium strategies and comment on their efficiency. Nonetheless, as we add more periods in the game, it becomes complicated to find all the existing equilibrium strategies, since even the method described before and used in the two period games becomes time-consuming. Nevertheless, the 3 period game and the 4 period game, where there exist $m = 8$ positive types (and 8 symmetric negative types and a 0 type) were analysed by using computer search methods, and the list below shows that in the case of the 4 periods, there exist at least 2 equilibrium strategies, namely

$s1 = ((2,2,2,2,2,3,3,4,4),(3,3,3,3,3,4,4,5,5),(4,4,4,5,5,5,6,7,8))$ and

$s2 = ((2,2,2,2,3,3,3,3,4),(3,3,3,3,3,4,4,5,5),(4,4,5,5,5,6,6,7,8))$.

This fact illustrates that the equilibrium in the discrete game may not be unique even in discrete type games of more than 2 periods. Thus, it becomes apparent that it is possible that there exist multiple equilibria in any number of periods n, even though this is not investigated thoroughly at this point and further research needs to be done.

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